Dimension Four Wins the Same Game as the Standard Model Group

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Abstract

In a previous article Don Bennett and I looked for, found and proposed a game in which the Standard Model Gauge *Group* $S(U(2) \times U(3))$ gets singled out as the "winner". Here I propose to extend this "game" to construct a corresponding game between different potential dimensions for space-time. The idea is to formulate, how the same competition as the one between the potential gauge groups would run out, if restricted to the potential Lorentz or Poincare groups achievable for different dimensions of space-time d. The remarkable point is that it is the experimental space-time *dimension 4 which wins.* So the same function defined over Lie groups seems to single out *both* the gauge group *and* the dimension of space time in nature. This seems a rather strange coincidence, unless there really is some similar physical reason behind causing our game-variable (or goal variable) to be selcted to be maximal. It has crudely to do with that the groups prefered are easily represented on very "small" but faithfull representations.

1 Introduction

The main idea of the present series of articles is to seek some game, that at the same time can select out the gauge group observed in nature - let us suppose we should have the Standard Model Group $S(U(2) \times U(3))$ in nature, say - and *also* the gauge group (whatever that means) of the (gravitational) general relativity by saying that nature has chosen the winner in this game. In the previous article[1] we sought in this way to invent a game or rather *a "goal quantity"* - which were at first the ratio C_A/C_F of the quadratic Cassimirs for the group in question for the adjoint representation to the quadratic Cassimir of some "small" but still faithful representation- in such a way that this ratio would take its largest value for the by nature chosen (gauge) group. Actually N. Brene and I had already earlier proposed another game that essentially pointed also to the Standard Model Gauge group being the winner [6], but it is the more recent proposal with the quadratic Cassimirs or rather their ratio C_A/C_F which we seek to generalize to determine the dimension of space time in this article. This concept of the gauge group for general relativity may be a bit imprecise, and so I want at first to simplify a little bit by making a few ad hoc decissions to extract a group that essentially is the gauge group of general relativity, even if we should not have chosen the definition of this concept completely clearly yet.

A first candidate, which is for me rather attracktive for the purpose, is simply the Lorentz group meaning the group of Lorentz boosts and rotations.

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You could consider the attitude of the present article and the foregoing in the series [1] as attempts to extract the information[7] discussed in the parameters or details of the Standard Model in order to use it for finding the hypotesized more fundamental theory beyond the Standard Model.

1.1 History of Explaining 3+1=4 Dimensions

As said the main purpose of the present article is to use the idea from our earlier article [1] to explain why nature should have chosen just 4 (meaning 3+1) space time dimensions. This way to be explained below is new relativ to earlier attempts to explain, why we shall just have four dimensions:

One of the earlier attempts is my own [9] starting the idea of "Random Dynamics" by pointing out that in a *non-Lorentz invariant* theory - being a quantum field theory in which neither rotational nor boost invariance is present, but only translational invariance - one finds genericly that assumming an appropriate Fermisurface an *effective Lorentz invariance with 3+1 dimesnions* appears automaticly! In this sense I claimed to derive under very general assumptions - as almost unavoidable - the appearance of both Lorentz and thereby rotational invariance and of just the right number of dimensions, 4=3+1. The success of this dimensionality post-diction [9] were for me the introduction to a long series of works seeking to derive from almost nothing or a random theory - not obeying many of the usual principles - which I gave the name "Random Dynamics" [9, 10, 11, 12, **?**], many of the known physical laws. Really we may consider the present work as an alternative attempt to derive the dimension of space time much in Random Dynamics way, in as far as we end up suggesting the philosophy that it would be most easy/likely to get just the experimental dimensionality by accident. (If sucessful then of course the present work would be a second derivation of 3+1 dimensions in Random Dynamics).

Also Max Tegmark has derived 3+1 dimensions from a similar Random Dynamics like philosophy of "all mathematics being realized"[2]. Max Tegmark considers the differential equations for the time development of fields so as to guarantee equations with predictivity, as well as the stability of atoms. For the anyway from his arguments needed case of just one time his figure shows that the field equation would be elliptic and thus unpredictable for d=1, too simple for d=2 and d=3, and unstable meaning unstable atoms say for d=5 or more. The latter point goes back to Ehrenfest in 1917 [3], who argued that neither atoms nor planatary systems could be stable in more than four space time dimensions.

Also known is the story that in say two spatial dimensions (corresponding to 3=2+1 space time dimensions) an animal - as ourselves - having an intestine channel would fall apart into two pieces. Thus by an antropic principle 3=2+1 should not be possible to have us.

According to a review of antropic questions by Gordon Kane [4]:

"One aspect of our universe we want to understand is the fact that we live in three space dimensions. There is an anthropic explanation. It was realized about a century ago[3] that planetary orbits are not stable in four or more space dimensions, so planets would not orbit a sun long enough for life to originate. For the same reason atoms are not stable in four or more space dimensions. And in two or one space dimensions there can be neither blood flow nor large numbers of neuron connections. Thus interesting life can only exist in three dimensions. Alternatively, it may be that we can derive the fact that we live in three dimensions, because the unique ground state of the relevant string theory turns out to have three large dimensions (plus perhaps some small ones we are not normally aware of). Or string theory may have many states with three space dimensions, and all of them may give universes that contain life".

Further one has considered the renormalizability of quantum field theories not being possible

for higher than 4 dimensions, except for the scalar ϕ^3 coupling theory, which is anyway not good [14].

In theories which like string theories or Norma Mankoc Borstnik's model [5] are Kaluza-Klein-like the question of understanding the effective dimension for long distances being 3 space plus 1 time dimension would a priori mean an understanding of why precisely there is that number of extra dimensions being somehow compactified that just three space dimensions survive as essentailly flat and exteended. In superstring theory the consistency requires fundamentally 9 +1 dimensions of space time. If one takes it that it is needed that the compact space described by the as extra dimensions appearing dimensions must be a Callaby-Yau spaace, then since the latter has 6 dimensions the observed or flat dimensions must make up (9+1) - (6+0) = 3+1. So the combination of the susperstring with the requirement of using Callaby-Yau compactification do indeed explain why we have just the experimental number of dimensions[15, 16].

In [17] you find:

"Now to make contact with our 4-dimensional world we need to compactify the 10-dimensional superstring theory on a 6-dimensional compact manifold. Needless to say, the Kaluza Klein picture described above becomes a bit more complicated. One way could simply be to put the extra 6 dimensions on 6 circles, which is just a 6-dimensional Torus. As it turns out this would preserve too much supersymmetry. It is believed that some supersymmetry exists in our 4-dimensional world at an energy scale above 1 TeV (this is the focus of much of the current and future research at the highest energy accelerators around the world!). To preserve the minimal amount of supersymmetry, N=1 in 4 dimensions, we need to compactify on a special kind of 6-manifold called a Calabi-Yau manifold. "

1.2 Development of Goal Quatities

In the present article we shall ignore these antropic principle arguments and seek to get a statement that the experimental number of dimensions just maximizes some quantity, that is a relativly simple function of the group structure of say the Lorentz group, and which we then call a "goal quantity".

Let me therefore list some of the first approximation simplified proposals which we suggest for this *goal quantity*. But this is for the dimension a two step procedure: 1) we first use the proposals in our previuos article [1] to give a number - a goal quantity - for any Lie group. 2) we have to specify on which group we shall take and use the procedure of previous work;shall it be the Lorentz group, its covering group or somehow an attempt with the Poincare group ? :

- a. Just take the Lorentz group and calculate for that the Dynkin index[8] or rather the quantity which we already used as goal quantity in the previous article [1] C_A/C_F. This gets especially simple for the except for dimension $d = 2$ or smaller semi-simple Lorentz groups (simple in the mathematical sense of not having any invariant nontrivial subgroup; semi-simple: no abelian invariant subgroup);since though the global structure of the Lorentz group is not fixed untill we assign it a meaning we are really here having in mind: The Lorentz group shall "have simple Lie algebra" . For simple groups we can namely ignore the minor corrections invented for the improvement in the case of an Abelian component present in the potential gauge group.

- b. We supplement in a somewhat ad hoc way the above *a.*, i.e. C_A/C_F by taking its $\frac{d+1}{d-1}$th power. The idea behind this proposal is that we think of the Poincare group instead of as under *a.* only on the Lorentz group part, though still in a crude way. This

means we think of a group, which is the Poincare group, except that we for simplicity ignore that the translation generators do not commute with the Lorentz group part. Then we assign in accordance with the ad hoc rule used in [1] the abelian sub-Lie- algebra a formal replacement 1 for the ratio of the quadratic Cassimirs C_A/C_f:I.e. we put "$C_A/C_F|''_{Abelian\ formal} = 1$. Next we construct an "average" averaged *in a logarithmic way* (meaning that we average the logathms and then exponentiate again) weighted with the dimension of the Lie groups over all the dimensions of the Poincare Lie group. Since the dimension of the Lorentz group for d dimensional space-time is $\frac{d(d-1)}{2}$ while the Poincare group has dimension$\frac{d(d-1)}{2} + d = \frac{d(d+1)}{2}$ the logarithmic averageing means that we get

$$exp(\frac{\frac{d(d-1)}{2}\ln(C_A/C_F)|_{Lorentz} + \ln(1) * d}{d(d+1)/2}) = (C_A/C_F)|_{Lorentz}^{\frac{d(d-1)}{2}/\frac{d(d+1)}{2}} = (C_A/C_F)|_{Lorentz}^{\frac{d-1}{d+1}} \tag{1}$$

That is to say we shall make a certain ad hoc partial inclusion of the abelian dimensions in the Poincare groups.

To be concrete we here propose to say crudely: let the poincare group have of course d "abelian" genrators or dimensions. Let the dimension of the Lorentz group be $d_{Lor} = d(d-1)/2$; then the total dimension of the Poincare group is $d_{Poi} = d + d_{Lor} = d(d+1)/2$. If we crudely followed the idea of weighting proposed in the previous article [1] as if the d "abelian" generators were just simple cross product factors - and not as they really are: not quite usual by not commuting with the Lorentz generators - then since we formally are from this previous article suggested to use the *as if number 1 for the abelian groups*, we should use the quantity

$$(C_A/C_F)|_{Lor}^{\frac{d_{Lor}}{d_{Poi}}} = (C_A/C_F)|_{Lor}^{\frac{d-1}{d+1}} \tag{2}$$

as goal quantity.

- c. We could improve the above proposals for goal quantities *a.* or *b.* by including into the quadratic Cassimir C_A for the adjoint representation also contributions from the translation generating generators, so as to define a quadratic Cassimir for the whole Poincare group. This would mean that we for calculating our goal quantity would do as above but

$$\textbf{Replace} : C_A \to C_A + C_V, \tag{3}$$

where C_V is the vector representation quadratic Casimir, meaning the representation under which the translation generators transform under the Lorentz group. Since in the below table we in the lines denoted "no fermions" have taken the "small representation" F to be this vector representation V, this replacement means that we replace the goal quantity ratio C_A/C_F like this:

$$\textbf{(S)O(d),} \quad \textbf{"no spinors":} \tag{4}$$
$$C_A/C_F = C_A/C_V \quad \to \quad (C_A + C_V)/C_F = C_A/C_F + 1 \tag{5}$$
$$\textbf{Spin(d),} \quad \textbf{"with spinors"}: \tag{6}$$
$$C_A/C_F \quad \to \quad (C_A + C_V)/C_F \tag{7}$$
$$= C_A/C_F + (C_A/C_V)^{-1}(C_A/C_F) \tag{8}$$
$$= (1 + (C_A/C_F)|^{-1}_{\text{no spinors}})C_A/C_F. \tag{9}$$

Let me stress though that this proposal c. is not quite "fair" in as far as it is based on the Poincare group, while the representations considered are <u>not</u> faithfull w.r.t. to the whole Poincare group, but only w.r.t. the Lorentz group

- d. To make the proposal *c.* a bit more "fair" we should at least say: Since we in *c.* considered a representation which were only faithfull w.r.t. the Lorentz subgroup of the Poincare group we should at least correct the quadratic Casimir - expected crudely to be "proportinal" to the number of dimensions of the (Lie)group - by a factor $\frac{d+1}{d-1}$ being the ratio of the dimension of the Poincare (Lie)group, $d + d(d-1)/2$ to that of actually faithfully represented Lorentz group $d(d-1)/2$. That is to say we should before forming the ration of the improved C_A meaning $C_A + C_V$ (as calculated under *c.*) to C_F replace this C_F by $\frac{d+1}{d-1} * C_F$, i.e. we perform the replacement:

$$C_F \to C_F * \frac{d(d-1)/2 + d}{d(d-2)/2} = C_F * \frac{d+1}{d-1}. \tag{10}$$

  Inserted into $(C_A + C_V)/C_F$ from *c.* we obtain for the in this way made more "fair" approximate "goal quantity"

$$\text{“goal quantity”}|_{\text{no spinor}} = (C_A/C_F + 1) * \frac{d-1}{d+1} \tag{11}$$

$$\text{“goal quantity”}|_{\text{w. spinor}} = (1 + (C_A/C_F)|^{-1}_{\text{no spinor}}) * C_A/C_F * \frac{d-1}{d+1} \tag{12}$$

  This proposal d. *should then at least be crudely balanced with respect to how many dimensions that are represented faithfully.*

The reader should consider these different proposals for a quantity to maximize (= use as goal quantiy) as rather closely related versions of a quantity suggested by a perhaps a bit vague idea being improved successively by treating the from our point of view a bit more difficult to treat Abelian part (the translation part of the Poincare group) at least in an approximate way. One should have in mind, that this somewhat vague basic idea behind is: The group selected by nature is the one that counted in a "normalization determined from the Lie algebra of the group" can be said to have a faithfull representation (F) the matrices of which move as little as possible when the group element being represented move around in the group.

Let me at least clarify a bit, what is meant by this statement:

We think by representations as usual on linear representations, and thus it really means representation of the group by means of a homomorphism of the group into a group of matrices. The requirement of the representation being faithful then means, that this group of matrices shall actually be an isomorphic image of the original group. Now on a system of matrices we have a natural metric, namely the metric in which the distance between two matrices **A** and **B** is given by the square root of the trace of the numerical square of the difference

$$dist = \sqrt{tr((\mathbf{A} - \mathbf{B})(\mathbf{A} - \mathbf{B})^{+})}. \tag{13}$$

To make a comparizon of one group and some representation of it with another group and its representation w.r.t. to how fast the representation matrices move for a given motion of the group elements we need a normalization giving us a weldefined metric on the groups w.r.t. which we can ask for the rate of variation of the representations. In my short statement I suggested that this "normalization should be determined from the Lie algebra of the group". This is to be

taken to mean more precisely, that one shall consider the *adjoint* representation, which is in fact completely given by the Lie algebra, and then use the same distance concept as we just proposed for the matrix representation $\sqrt{tr((\mathbf{A}-\mathbf{B})(\mathbf{A}-\mathbf{B})^{+})}$. In this way the quantity to minimize would be the ratio of the motion-distance in the representation - F say - and in the Lie algebra representation - i.e. the adjoint representation. But that ratio is just for infinitesimal motions $\sqrt{C_F/C_A}$. So if we instead of talking about what to minimize, inverted it and claimed we should maximize we would get $\sqrt{C_A/C_F}$ to be *maximized*. Of course the square root does not matter, and we thus obtain in this way a means to look at the ratio C_A/C_F as a measure for the motion of an element in the group compared to the same element motion on the representation.

It might not really be so wild to think that a group which can be represented in a way so that the representation varies little when the group element moves around would be easier to get realized in nature than one that varies more. If one imagine that the potential groups become good symmetries by accident, then at least it would be less of an accident required the less the degrees of freedom moves around under the group to have the symmetry (approximately). It is really such a philosophy of it being easier to get some groups approximately being good symmetries than other, and those with biggest C_A/C_F should be the easiest to become good symmetries by accident, I argue for. That is indeed the speculation behind the present article as well as the previous one [1] that symmetries may appear by accident(then perhaps being stregthened to be exact by some means [10, 18]).

But let us stress that you can also look at the present work and the previous one in the following phenomenological philosophy:

We wonder, why Nature has chosen just 4 (=3+1) dimesnions and why Nature - at the present experimentally accessible scale at least - has chosen just the Standard Model group $S(U(2)\times U(3))$? Then we speculate that there might be some quanity charactrizing groups, which meassures how well they "are suited " to be the groups for Nature. And then we begin to *seek* that quantity as being some function defined on the class of abstract groups - i.e. giving a number for each abstract (Lie?) group - of course by proposing for ourselves at least various versions or ideas for what such a *relatively simple* function defined on the abstract Lie groups could be. Then the present works - this paper and the previous one - represents the present status of the search: We found that with small variations the types of such functions representing the spirit of the *little motion of the "best" faithful representation*,i.e. essentially the largest C_A/C_F, turned out truly to bring Natures choises to be (essentilly) the winners.

In this sense we may then claim that we have found phenomenologically that at least the "direction" of a quantity like C_A/C_F or light modifications of it is a very good quantity to make up a "theory" for why we have got the groups we got!

1.3 Outline

In the following section 2 we review the main results of the C_A/C_F quantity, which we in the previus article studied for the various Lie groups in order to discuss, that the Standard Model group could be made to be favoured. In section 3 we then extract and concentrate on those groups which can be Lorentz groups. The main content of both these sections are actually the tables listing the results of the proposed to be maximized quantities for the relevant groups. In section 4 we resume and conclude, that actually we may be on the track to have found a *common* reason or explanation for, that we have 3 +1 dimensions, and for the gauge group of the Standard Model!

2 Our Previous Numbers

In the previous work by D. Bennett and myself [1] we essentially collected the ratios (related to Dynkin index [8]) C_A/C_F, where we for the representation of the group in question G selected that representation F, which would give the largets value for this ratio C_A/C_F. (In the table we give in a few cases two proposals for F, but really it is what one would loosely call the smallest faithful representation). But we shall have in mind that this ratio is only welldefined for the simple Lie groups; and it is thus only for the simple groups we could make a clean table as the one just below. For semi-simple Lie groups it is strictly speaking needed to specify a replacement quantity, that can be the needed generalization to semisimple Lie groups. One shall naturally construct a logarithmic average weighted with the dimensions of the various simple group factors contained in the semisimple Lie group written as a product of its simple invariant subgroups. Extending the generalization even to inclusion of $U(1)$-factors in the Lie group gets even a bit more arbitrary, but we did choose the rule of counting the $U(1)$ factors as, if they had C_A/C_F equal to unity. The problem with the $U(1)$'s, the Abelian groups, is that the adjoint quadratic Cassimir C_A is just zero and does not provide a good normalization. But although we thus have to declare formally C_A/C_F to be unity at first for $U(1)$ we take the opportunity to - and we think it is very natural - to include a correction depending not only on the Lie-*algebra* but also on the *group* structure in a way roughly describing that a $U(1)$ representation with a small "charge" is "smaller" than one with a lrager charger charge in a very similar way to the way in which a small quadratic Cassimir signals a "small" representation.

Our Ratio of Adjoint to "Simplest" (or smallest) Quadratic Casimirs C_A/C_F

$$\frac{C_A}{C_F}|_{A_n} = \frac{2(n+1)^2}{n(n+2)} = \frac{2(n+1)^2}{(n+1)^2-1} = \frac{2}{1-\frac{1}{(n+1)^2}} \tag{14}$$

$$\frac{C_A}{C_{F\ vector}}|_{B_n} = \frac{2n-1}{n} = 2-\frac{1}{n} \tag{15}$$

$$\frac{C_A}{C_{F\ spinor}}|_{B_n} = \frac{2n-1}{\frac{2n^2+n}{8}} = \frac{16n-8}{n(2n+1)} \tag{16}$$

$$\frac{C_A}{C_F}|_{C_n} = \frac{n+1}{n/2+1/4} = \frac{4(n+1)}{2n+1} \tag{17}$$

$$\frac{C_A}{C_{F\ vector}}|_{D_n} = \frac{2(n-1)}{n-1/2} = \frac{4(n-1)}{2n-1} \tag{18}$$

$$\frac{C_A}{C_{F\ spinor}}|_{D_n} = \frac{2(n-1)}{\frac{2n^2-n}{8}} = \frac{16(n-1)}{n(2n-1)} \tag{19}$$

$$\frac{C_A}{C_F}|_{G_2} = \frac{4}{2} = 2 \tag{20}$$

$$\frac{C_A}{C_F}|_{F_4} = \frac{9}{6} = \frac{3}{2} \tag{21}$$

$$\frac{C_A}{C_F}|_{E_6} = \frac{12}{\frac{26}{3}} = \frac{18}{13} \tag{22}$$

$$\frac{C_A}{C_F}|_{E_7} = \frac{18}{\frac{57}{4}} = \frac{72}{57} = \frac{24}{19} \tag{23}$$

$$\frac{C_A}{C_F}|_{E_8} = \frac{30}{30} = 1 \tag{24}$$

For calculation of this table seek help in[20, 19]

In the just above table we have of course used the conventional notation for the classification of Lie algebras, wherein the index n on the capital letter denotes the rank of the Lie algebra, and:

- A_N is $SU(n+1)$,
- B_n are the symplectic Lie algebras.
- C_n is the odd orthogonal Lie algebra for $SO(2n+1)$ or for its covering group Spin(2n+1),
- D_n is the even dimension orthogonal Lie algebra for $SO(2n)$ or its covering group $Spin(2n)$,
- while F_4, G_2, and E_n for $n = 6, 7, 8$ are the exceptional Lie algebras.

The words *spinor* or *vector* following in the index the letter F which itself denotes the "small" representation - i.e. most promissing for giving a small quadratic Cassimir C_F - means that we have used F respectively the sinor and vector representation.

It may be reasuring to check that our goal quantity for the simple groups C_A/C_F becomes the same for the cases of isomorphic Lie algebras such as $B_2 \cong C_2$ and $C_3 \cong A_3$. .

3 Competition Among Lorentz Groups on C_A/C_F and the Like

Caption: In the below tables we evaluate for different dimensions d of the Minkowski space-time - for simplicity here replaced by the Euklideanized d dimensional space-time, but that makes no difference for our calculation here - the first two goal-quantities proposed, *a.* and *b.* in subsection 1.2 written in respectively 5th and 7th columns. Because of the ambiguity of the global structure of the Lorentz-**group** the group in d dimension may be either O(d) (essentially SO(d) if we do not include parity) or Spinor(d) if we use the covering group. We have therefore for each value of the dimension d two items corresponding to these two global extensions of the Lie algebra of the Lorentz group, and they are denoted by "no spinors" and "with spinors" respectively.

Dimension	Lorentz group	Spinor or not	Rank	Ratio C_A/C_F max a)	$\frac{d_{Lor}}{d_{Poi}}$	$(C_A/C_F)^{\frac{d_{Lor}}{d_{Poi}}}$ max b)
d=1	discrete		0	-	0	indefinite 0^0
d=2[1]	(S)O(2) = U(1)	no spinor	1	- (formally 1)	1/3	- (formally 1)
	U(1)	with spinor	1	-(formally 2)	1/3	-(formally $2^{1/3} = 1.26$)
d=3	(S)O(3)	no spinor	1	1	1/2	1
	Spin(3)=SU(2)	with spinor	1	8/3= 2.6667	1/2	$\sqrt{8/3}$ =1.632993162
d=4	(S)O(4)	no spinor	2	4/3=1.3333	3/5	$(4/3)^{3/5}$ =1.188401639
	Spin(4) $= SU(2) \times SU(2)$	with spin	2	8/3=2.6667	3/5	$(8/3)^{3/5}$ =1.801280051
d=5	(S)O(5)	no spinor	2	3/2=1.5	2/3	$(3/2)^{2/3}$ =1.310370697
	Spin(5)	with spinor	2	12/5=2.4	2/3	$(12/5)^{2/3}$ =1.792561899
d=6	(S)O(6)	no spinor	3	8/5=1.6	5/7	$(8/5)^{5/7}$ = 1.398942897
	Spin(6) = SU(4)	with spinor	3	32/15=2.1333	5/7	$(32/15)^{5/7}$ =1.718074304
d=7	(S)O(7)	no spinor	3	13/7=1.8571	3/4	$(13/7)^{3/4}$ =1.590867407
	Spin(7)	with spinor	3	40/21=1.9048	3/4	$(40/21)^{3/4}$ =1.621363987
d=8	(S)O(8)	no spinor	4	12/7=1.7143	7/9	$(12/7)^{7/9}$ =1.520774129
	Spin(8)	with spinor	4	12/7=1.7143	7/9	$(12/7)^{7/9}$ =1.520774129
d=9	(S)O(9)	no spinor	4	7/4=1.75	4/5	$(7/4)^{4/5}$ =1.564697681
	Spin(9)	with spinor	4	14/9=1.5556	4/5	$(14/9)^{4/5}$ = 1.423994858
d=10	(S)O(10)	no spinor	5	16/9=1.7778	9/11	$(16/9)^{9/11}$ =1.601198613
	Spin(10)	with spinor	5	64/45=1.4222	9/11	$(64/45)^{9/11}$ =1.33399805
d=11	(S)O(11)	no spinor	5	9/5=1.8	5/6	$(9/5)^{5/6}$ =1.632026054
	Spin(11)	with spinor	5	72/55= 1.3091	5/6	$(72/55)^{5/6}$ =1.251626758
d=12	(S)O(12)	no spinor	6	44/23= 1.9130	11/13	$(44/23)^{11/13}$ =1.731340775
	Spin(12)	with spinor	6	40/33= 1.2121	11/13	$(40/33)^{11/13}$ =1.176773318
d=13	(S)O(13)	no spinor	6	25/13 =1.9231	6/7	$(25/13)^{6/7}$ 1.75156277
	Spin(13)	with spinor	6	44/39 =1.1282	6/7	$(44/39)^{6/7}$ =1.108929813
d=14	(S)O(14)	no spinor	7	24/13 =1.8461	13/15	$(24/13)^{13/15}$ = 1.701239682
	Spin(14)	with spinor	7	104/105 =0.9905	13/15	$(104/105)^{13/15}$

Dimension	Lorentz group	Spinor or not	Rank	Ratio C_A/C_F max a)	$\frac{d_{Lor}}{d_{Poi}}$	$(C_A/C_F)^{\frac{d_{Lor}}{d_{Poi}}}$ max b)
d odd	(S)O(d)	no spinor	n =(d-1)/2	2- 1/n =2 -2(d-1)	$\frac{d-1}{d+1}$	$(2-\frac{1}{d-1})^{\frac{d-1}{d+1}}$
	Spin(d)	with spinor	n=(d-1)/2	$\frac{8(2n-1)}{n(2n+1)}$ $=\frac{16(d-2)}{d(d-1)}$	$\frac{d-1}{d+1}$	$(\frac{16(d-2)}{d(d-1)})^{\frac{d-1}{d+1}}$
d even	(S)O(d)	no spin	n=d/2	$\frac{4(n-1)}{2n-1}$ $=\frac{2(d-2)}{d-1}$	$\frac{d-1}{d+1}$	$(\frac{2(d-1)}{d-1})^{\frac{d-1}{d+1}}$
	Spin(d)	with spinor	n=d/2	$\frac{16(d-2)}{d(d-1)}$	$\frac{d-1}{d+1}$	$(\frac{16(d-2)}{d(d-1)})^{\frac{d-1}{d+1}}$
d $\rightarrow \infty$		no spinor	$\rightarrow \infty$	$\rightarrow 2$	$\rightarrow 1$	$\rightarrow 2$
		with spinor	$\rightarrow \infty$	$\rightarrow 0$	$\rightarrow 1$	$\rightarrow 0$

3.1 Discussion of Table

Motivated either by the fact that we have spinor transforming particles in nature - namely the Fermions - or because the goal-numbers for the spinor groups are anyway the biggest (most competitive) we should think of the Lorentz *group* as the spinor group and therefore in the above table read the Spin(d) entrances rather than the $(S)O(d)$-entrances:

Concentrating on the $Spin(d)$-entrances we then find that with the proposal *a.* of subsection 1.2 the dimensions d = 3 and d= 4 stand even with the same goal-number 8/3 = 2.6667. But note that at least the experimental dimension 4 already is in the sample of the “winners” with the simple choice of *a.* meaning, that we only consider the genuine Lorentz group - but totally ignore the abelian part of the Poincare group.

Next when we go to the slightly more complicated version of a goal-quantity namely *b.* we get the sepeation between also d=3 and d=4, and it is the d=4 dimension, that “wins”, because we get for d=3 only 1.6330, while we for d=4 obtain 1.8013. Thus in this approximate treatment of the Abelian part also being included the (after all rather) “little” difference between the two schemes *a.* and *b.* leads to giving the d=4 case - the experimental case the little push forward making the experimental dimension d=4 be the only winner!

Caption: We have put the goal-numbers for the third proposal c in which I (a bit more in detail) seek to make an analogon to the number used in the reference [1] in which we studied the gauge group of the Standard Model. The purpose of *c.* is to approximate using the *Poincare group* a bit more detailed, but still not by making a true representation of the Poincare group. I.e. it is still not truly the Poincare group we represent faithfully, but only the Lorentz group, or here in the table only the covering group $Spin(d)$ of the Lorentz group. However, I include in the column marked “*c.*, max c)” in the quadratic Cassimir C_A of the Lorentz group an extra term comming from the structure constants describing the non-commutativity of the Lorentz group generators with the translation generators C_V so as to replace C_A in the starting expression of ours C_A/C_F by $C_A + C_V$. In the column marked “*d.*, max d) ” we correct the ratio to be more “fair” by counting at least that because of truly faithfully represented part of the Poincare group in the representations, I use, has only dimesnion $d(d-1)/2$ (it is namely only the Lorentz group) while the full Poincare group - which were already in *c.* but also in *d.* used in the improved C_A being $C_A + C_V$ - is $d(d-1)/2 + d = d(d+1)/2$. The correction is crudely made by the dimesion ratio $dim(Lorentz)/dim(Poincare) = (d-1)/(d+1)$ given in the next to last column.

Dimension	Lorentz group (covering group)	Ratio C_A/C_F for spinor	Ratio C_A/C_V as "no spinor"	$c.$-quantity max c)	$\frac{d-1}{d+1}$	$d.$-quantity max d)
2^2	U(1)	-(formally 2)	-(formally 1)	4	1/3	4/3=1.33
3	spin(3)	$\frac{8}{3} = 2.6667$	1	$\frac{16}{3} = 5.3333$	$\frac{2}{4} = .5$	$\frac{8}{3} = 2.6667$
4	$Spin(4)$ $= SU(2) \times SU(2)$	$\frac{8}{3} = 2.6667$	$\frac{4}{3}$	$\frac{14}{3} = 4.6667$	$\frac{3}{5} = .6$	$\frac{14}{5} = 2.8$
5	Spin(5)	$\frac{12}{5} = 2.4$	$\frac{3}{2} = 1.5$	4	$\frac{4}{6} = .667$	$\frac{8}{3} = 2.6667$
6	$Spin(6)$	$\frac{32}{15}$	$\frac{8}{5} = 1.6$	$\frac{52}{15} = 3.4667$	$\frac{5}{7} = .714$	$\frac{52}{21} = 2.4762$
d odd	Spin(d)	$\frac{8(2n-1)}{n(2n+1)}$ $= \frac{16(d-2)}{d(d-1)}$	$2 - 1/n$ $= 2 - 2/(d-1)$	$\frac{8(3d-5)}{d(d-1)}$	$\frac{d-1}{d+1}$	$\frac{8(3d-5)}{d(d+1)}$
d even	$Spin(d)$	$\frac{16(d-2)}{d(d-1)}$	$\frac{4(n-1)}{2n-1} = \frac{2d-4}{d-1}$	$\frac{8(3d-5)}{d(d-1)}$	$\frac{d-1}{d+1}$	$\frac{8(3d-5)}{d(d+1)}$
d odd $\to \infty$	Spin(d)	$\approx 16/d$	$\to 2$	$\approx 24/d$	$\to 1$	$\approx 24/d \to 0$
d even $\to \infty$	Spin(d)	$\approx 16/d$	$\to 2$	$\approx 24/d$	$\to 1$	$\approx 24/d \to 0$

We see from the table here for simplicity made only for the most competitive case of "with spinor" in the terminology the foregoing table that with column $c.$-goal numbers actually it is d=3 rather than the experimental dimension d=4 that "wins"(i.e. is largest), but this series of proposed numbers $c.$ is not truly "fair" in as far as one has effectively used only the Lorentz group in the denominator C_F but at least crudely the full Poincare group in the numerator $C_A + C_V$. Thus in order to avoid a simple wrong expected variation of a quadratic Cassimir with the dimensionality of the Lie group, we should at least correct the denominator C_F by multiplying it by the ratio of the dimension of the Poincare Lie group over that of the Lorentz Lie group, $(d+1)/(d-1)$. When we make this "fairness correction" at least crudely getting no obvious wrong Lie-group-dimension-dependent factor in, then the dimension d=4 becomes (again) the winner! In fact we get for d=4 (the experimental dimension) the goal quantity in column $d.$ equal to 14/5= 2.8 while accidentally the two neighboring dimensions d=3 and d=5 both gets in stead $8/3 = 2.66667$, which is less.

But notice that it is a rather smooth peaked curve with the peak near the experimental dimesnion 4, so that the latter becomes the winner among integers, but it is only by a tiny bit it wins. That is to be expected from the smoothness of the variation of the goal number with the dimension d. This smallness of the excess making the d=4 be the winner of course makes the uncertainty bigger and my "crude " corrections rather than exactly calculating some welldefined quantity is thus not so convincing. Anyway I think, that the accuracy may be good enough, and the simplicity of the proposed goal quantities sufficient to make it at least highly suggestive, that the coincidence of the winning dimension and the experimental one means that we are on the right track!

4 Conclusion

We have found that a couple of very reasonable specifications of what the extention of our previous [1] quantity to be maximized to obtain the Standard Model gauge group leads to that the maximization of the generalized quantity gives as the "winner" the dimension d=4 as is empirically the dimension! That is to say we have found a possible *explanation* for, why we have just 4 (meaning 3+1) dimensions of space-time!

In fact we have extended the main idea of claiming the maximization of essentially the to the Dynkin index related group dependent quantity C_A/C_F (with C_A and C_F being the quadratic

Casimirs for respectively the adjoint representation C_A and for that (essentially) faithful representation F chosen so as to maximize the ratio C_A/C_F.) to lead to the experimentally realized group (the Standard Model group). For honesty it should be admitted that the victory of exactly the Standard Model Group were dependent on our slightly ad hoc treatment of the Abelian invariant subgroup - i.e. the $U(1)$ - needed because of the ratio C_A/C_F being rather meaningless a priori for an abelian group. For honesty we must also admit that in reference [1] we sneaked in a dependence on the *group* rather than only *Lie algebra* by considering the volume of the *group* which depends on the identification of center elements, properties being revealed phenomenolgically via the representations of the gauge group (except for the case of d=2 this detail is though not relevant for the Lorentz group and thus the dimension).

Our extension consists in that we in stead of the gauge group rather consider the *Lorentz group*, or we even seek to use the *Poincare group*. Then of course our quantity C_A/C_F or slight modifications/"improvements" of it - enumerted *a., b., c., d.* - will depend on the dimension d of space time.The dimension d gives of course a different Lorentz group for each value of d. We then inserted this d-dependent Lorentz group in stead of the gauge group which were studied in last paper[1]. The various modifications, *a., b., c., d.*, shall be considered attempts to use the *Poincare group* instead of the Lorentz group, but rather than truly doing that, make some approximate treatment as if crudely using the Poincare group.

The results of the search for the dimension having the largest "goal-quantity", using various proposals for the exact form such as *a.* meaning C_F/C_F simply, are the following:

- a. The simple quantity C_A/C_F for the *Lorentz group* with the same formal assignment for abelian group as used in [1], here making d=2 noncompetitive(but at least having a score formally), leads to d=3 and d=4 standing even, both scoring the same number $C_A/C_F = 8/3$.

- b. Making a crude correction to consider instead the quantity $(C_A/C_F)^{(d-1)/(d+1)}$ leads to that the experimental dimension of space time d=4 gets the largest score. The meaning of this slight modification of *a.* is that we make an attempt to take the group to replace the gauge group in our previous paper[1] to be the Poincare group rather than the Lorentz group. We, however, only make a crude attempt in that direction. Since the Poincare has the translation subgroups which are by themselves Abelian we naturally tend to use formally - just like in reference [1] - to assign a factor 1 to the abelian groups. Then we average in the logarithm our goal numbers for the various factors into which the group falls weighting with the dimension in the Lie algebra.

  The inclusion of the Poincare group is not done in a fully correct way though in as far as we only consider the faithfull representations of the Lorentz group and only extend a bit speculatively to weight as if we had the Poincare group.

- c. Still thinking of crudely using the Poincare group rather than the Lorentz group, we proposed to still take a representation F only of the Lorentz group, but evaluating the quadratic Cassimir for the Poincare group, although that sounds not quite "fair". The quadratic Cassimir we used here under *c.* for "the Poincare group" were taken to $C_A + C_V$, where V denotes the vector representation and thus C_V its quadratic Cassimir. In this "unfair" game the dimension for space time d=3=2+1 got the highest score. So our hoped for victory of the experimental dimension failed in this "unfair" proposal. *But since I stress the "unfairness" of this proposal, we should not take this proposal seriously.*

- d. This last proposal in the present article is a crude attempt at least to correct for, that the ratio of the dimensions of the Poincare and the Lorentz Lie groups is space-time

dimension d dependent. That is to say, we argue that the quadratic Cassimir C_F for the representation F of the Lorentz group should at least be scaled so as to correspond to a representaion rather of the Poincare group by being multiplied by the ratio of the Lie group dimensions of the Poincare group relative to that of the Lorentz group, $\frac{d(d-1)/2+d}{d(d-1)/2} = \frac{d+1}{d-1}$. That is to say we perform the crude correction of replacing

$$C_F \to C_F * \frac{d+1}{d-1}. \tag{25}$$

Since the quantity C_F occurs in the denomiator of the quantity $(C_F + C_V)/C_F$ maximized under *c.* of course this quantity is scaled the opposite way, and the goal-quantity in this proposal *d.* is taken as

$$\text{“goal quantity } d.\text{”} = \frac{(C_A + C_V)(d-1)}{C_F(d+1)}. \tag{26}$$

Now the result becomes that *the experimental dimension d=4 has the largest value for the goal quantity d.*, in as far as it gets

$$\frac{(C_A + C_V)(d-1)}{C_F(d+1)}|_{d=4} = \frac{14}{5} = 2.8, \tag{27}$$

while by accident the two neighboring space time dimensions 3 and 5 score only $\frac{8}{3} = 2.6667$. So indeed the *experimental space time dimension 4 won the most developped suggestion d.*.

This means that apart from the “unfair” proposal c., all the four proposals here have the space time dimension d=4 realized in nature obtain a largest “goal quantity” among the winners! In a. d=3 and d=4 share the winner place, but in the two other “fair” proposals *b.* and *d.* it is indeed space time dimension d=4, the experimental one, that gives the highest “goal quantity”.

Taking this result serious, and not as being just accidental coincidence or a result of inventive construction, it must tell us about the reason for that the dimensionality became just d=4. Taking our result serious we must look for what is the spirit behind the proposals above, so as to obtain an answer to “Why did we get just d=4 space time dimensions?”. This “spirit” behind the proposals here set up to select the experimental dimension d=4 seems to be that the group - the Poincare or Lorentz group or the gauge group say - should be *representable in a way where the matrices or other objects on which the group is represented are relatively slowly varying under the group.*

We may, if taking this “slowness” of the motion of the representative in the representation with the group element serious, seek to invent a model behind the 4-dimensional space time and the Standard Model gauge group that could explain that slowness. One possible such explanation could be:

Truly the fundamental physics model or theory is “random” and that without the symmetries we seek to explain. Then “by accident” there appears approximately some symmetries - and we here hope for say some Lorentz invariance symmetry -. Now we dream that there may be some way in which such an approximate symmetry can be automaticly become exact in practice. We, Førster, Ninomiya and me [10](see also Damgaard et al. [18]), have actually argued that gauge symmetry with electrodynamics (and Yang Mills theory) as example can occur in a whole phase in practice giving precisely the *massless* photon in that phase. Thus we can for symmetries that can somehow be considered gauge symmetry - as can the Lorentz symmetry in general relativity - speculate that such symmetries appear in practice as exact providided though, that

they are there approximately at first[22, 23].. But now the crux of the matter is that *if a symmetry is represented by slowly moving matrices or whatever, then one must expect that statistically it would be easier to get the symmetry approximately by accident.* If it were such that the fundamental theory could be considered random and only obtaining some symmetries by accident - at first approximately, but perhaps made exact by some mechanism[10, 22, 23, 18] - then we could consider the practically random Lagrange or action as taking random values for regions of some (small) size in the value space for the *representaion* of the group which gives the transformation properties of the fields or degrees of freedom under the group in question. Now when a group is represented by a representation, which in some sense is the represented matrix or field, and these fields or matrices move slowly for an appropriately normalized motion of the group element represented, then one can vary the group element much before one varies the representation field much. But this means that one needs less good luck to get a symmetry accidentally the slower the represention moves, because the displacement inside the group (itself) corresponding to one of the (small) size regions (over which we assume essential constancy of the action) becomes bigger the slower the representaion motion rate.

The crucial point should be that one would with the in some sense random action have a better chanse to obtain by accident a certain symmetry, when this symmetry is represented on the fields or degrees of freedom by a "slowly moving representaion", so such a symmetry would more often occur by accident, if one thinks this random action way.

So when our various "goal quantities" favour the experimentally found gauge group and the dimension of space time, it means that the groups realized in nature are the ones that have the optimal chanse to come out of a random action model. This is so because these goal quantities being large means that the representation motion is slow.

So the message from the gauge group and the dimension is that such a random action philosophy is one possible mechanism behind the choice by nature of the gauge groups and dimension.

The idea of there being made in some sense a lot of attempts randomly of groups to be tested off could be said to have remiscense of the idea of a gaugeglass [21], which though rather means the action is random quenched randomly locally, but that the gauge group is given from the start; but the spirit is similar.

A priori one should speculate about possible other machineries that could explain that precisely our type of "goal quantities"; but at first it seems that the random action type of model allowing symmetries of the type with highest goal quantities is a good idea and very likely something like that could be the reason behind the choice by nature of the gauge group (of the Standard Model) and of the dimension.

In any case we have found a surprisingly simple principle - the maximization of our to each other rather closely related "goal quantities" - leading to both the gauge group of the Standard Model *and* the dimension of space time being 4.

Let me stress that a work of the present type and of [1] - finding a goal quantity leading to the realized groups - is an attempt to ask phenomenologically whether there is some signal in the details of the phenomenologically present theory that successively can give us hint(s) about the more fundamental theory behind the presently working Standard Model with its gauge group $S(U(2) \times U(3))$ and the seeming dimension 4.

5 Acknowledgement

I would like to thank the Niels Bohr Institute for allowing me status as professor emeritus and for economical support and Matjaz Breskvar support for visiting the Bled Conference "Beyond the

Standard Models" wherein the first paper[1] in the present series of two articles were presented together with Don Bennett - although a year earlier though -, who is also thanked together with colleages discussing there the previous work.

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